软件分析与架构设计

指针分析

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指针分析（Pointer Analysis）

- **指针分析计算指向关系：Pts: Var $\rightarrow$ $\mathcal{P}$(OBJs)**
  - $Pts(x)$: 确定指针变量x运行时的可能取值$(may-points-to)$
  - **Var**：程序中的指针变量；**OBJs**：allocation site抽象，以表示从同一个分配点创建的所有运行时对象
  - *Must-points-to*关系不在考虑范围内
  - 指向关系可以用来计算别名(alias)关系
    - Alias(x, y) if and only if Pts(x) ∩ Pts(y) ≠ ∅
  - **计算收集语义**
- **限定需要分析的3地址IR指令：**
  - 暂时只支持流不敏感（flow-insensitive）分析，不考虑goto语句
  - 暂时只涉及过程内（intra-procedural）分析，不考虑call语句

$$Inst ::= x := y \mid x := \&p \mid x := *p \mid *p := x \mid x := y.f \mid x.f := y \mid p := q \mid x := malloc()$$

指针分析（Pointer Analysis）

❑例子：

	Strong Update	Weak Update
1. q := malloc() // O1	Pts(q) = {O1}	Pts(q) = {O1}
2. p := malloc() // O2	Pts(p) = {O2}	Pts(p) = {O2}
3. p := q	Pts(p) = {O1}	Pts(p) = {O1, O2}
4. r := &p	Pts(r) = {&p}	Pts(r) = {&p}
5. s := malloc() // O3	Pts(s) = {O3}	Pts(s) = {O3}
6. *r := s	Pts(p) = {O3}	Pts(p) = {O1, O2, O3}
7. t := &s	Pts(t) = {&s}	Pts(t) = {&s}
8. u := *t	Pts(u) = {O3}	Pts(u) = {O3}

流不敏感分析不区分指令执行顺序，只有弱更新，没有强更新

问题：如何设计一个指向分析算法？

收集语义（Collecting Semantics）

The **collecting semantics** requires us to know each execution of the program, assuming a (possibly infinite) trace for each run.

❑Infinite Domain value

$x \mapsto \{-\infty, \cdots, -2, -1, 0, 1, 2, \cdots + \infty\}$

```
1. Read x
2. y = 2 * x
3. z = 0
4. if y > 0 goto 6
5. z = 2 * x - y
6. ...
```

❑Infinite execution path

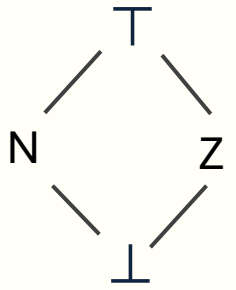
```
1. read x
2. t = 0
3. if x < 0 goto 7
4. x = x - 1
5. t = t + x
6. goto 3
7. print t
```

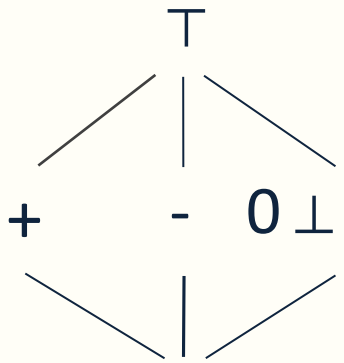
循环次数取决于输入x

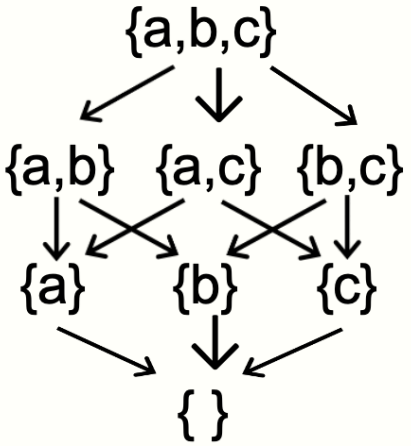
❑ 需要对数据域和控制流等进行抽象

抽象域（Abstraction）

- **零分析（Zero Analysis）**
  - 具体域：$\mathbb{Z}$：$\{-\infty, \cdots, -2, -1, 0, 1, 2, \cdots + \infty\}$
  - 抽象域：$\{\bot, \mathrm{N}, \mathrm{Z}, \top\}$
- **符号分析 (Sign Analysis)**
  - 具体域：$\mathbb{Z}$：$\{-\infty, \cdots, -2, -1, 0, 1, 2, \cdots + \infty\}$
  - 抽象域：$\{\bot, +, -, 0, \top\}$
- **指针分析（Pointer Analysis）**
  - 具体域：运行时堆对象或栈上位置
  - 抽象域：&t, alloca, 或new A对应行号表示
    - Allocation site abstraction
- **抽象域元素往往构成格（lattice）**



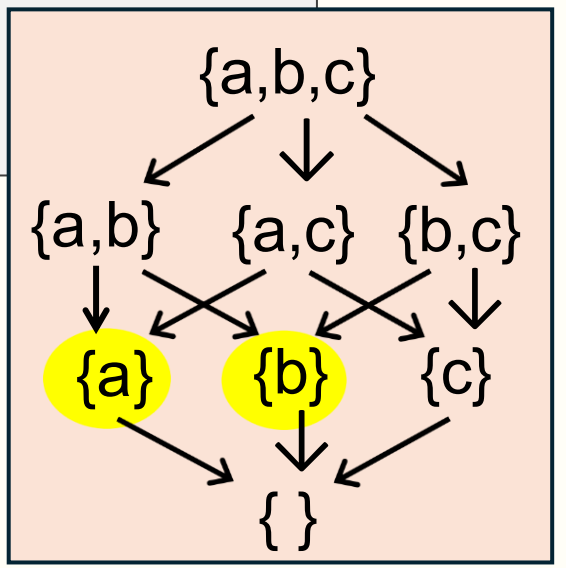




偏序（Partial orders）

Given a set S, a partial order $\sqsubseteq$ is a binary relation on S that satisfies:

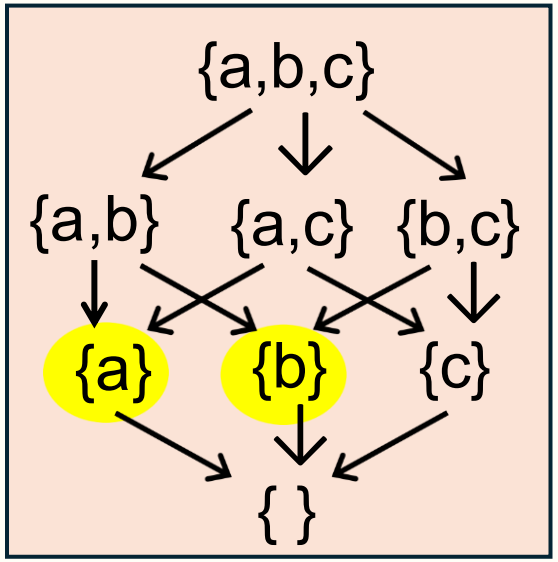
- reflexivity: $\forall x \in S: x \sqsubseteq x$
- transitivity: $\forall x, y, z \in S: x \sqsubseteq y \wedge y \sqsubseteq z \rightarrow x \sqsubseteq z$
- anti-symmetry: $\forall x, y \in S: x \sqsubseteq y \wedge y \sqsubseteq x \rightarrow x = y$



- $(S, \sqsubseteq)$**称为偏序集（poset）**
  - 一个集合的幂集和$\subseteq$（subset）构成一个偏序集
  - 偏序集中的两个元素可能无法比较,e.g., 右图中的{a}和{b}
- **Upper and Lower bounds**

Let $X \subseteq S$ be a subset, we say that $y \in S$ is an **upper bound** ($X \sqsubseteq y$) when $\forall\, x \in X: x \sqsubseteq y$;
We say that $y \in S$ is a **lower bound** ($y \sqsubseteq X$) when $\forall\, x \in X: y \sqsubseteq x$;
A ***least upper bound*** $\bigsqcup X$ is defined by $X \sqsubseteq \bigsqcup X \wedge \forall\, y \in S: X \sqsubseteq y \Rightarrow \bigsqcup X \sqsubseteq y$;
A ***greatest lower bound*** $\bigsqcap X$ is defined by $\bigsqcap X \sqsubseteq X \wedge \forall\, y \in S: y \sqsubseteq X \Rightarrow y \sqsubseteq \bigsqcap X$.

格（Lattices）



完全格

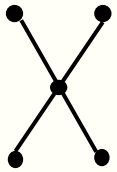
❑格与半格

- Given a poset (S, ⊑), ∀a, b ∈ S,
- if $a \sqcup b$ exists, then (S, ⊑) is called a join semilattice;
- if $a \sqcap b$ exists, then (S, ⊑) is called a meet semilattice;
- if both exist, then (S, ⊑) is called a lattice.

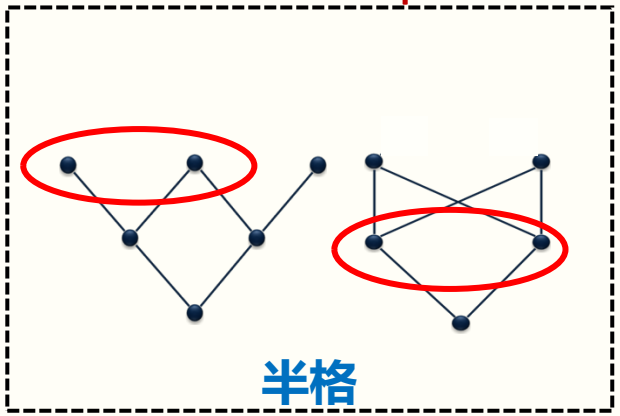
❑完全格

- If ∀ X ⊆ S, ⨅X and ⨆X exist, (S, ⊑) is called a complete lattice.
- A complete lattice must have a unique largest element ⊤ = ⨆S and a unique smallest element ⊥ = ⨅S
- A finite lattice is complete if ⊤ and ⊥ exist （一般有限格都是完全格）

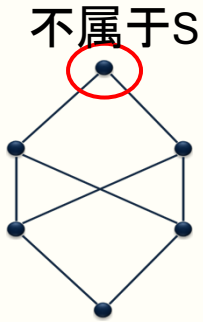
❑举例：



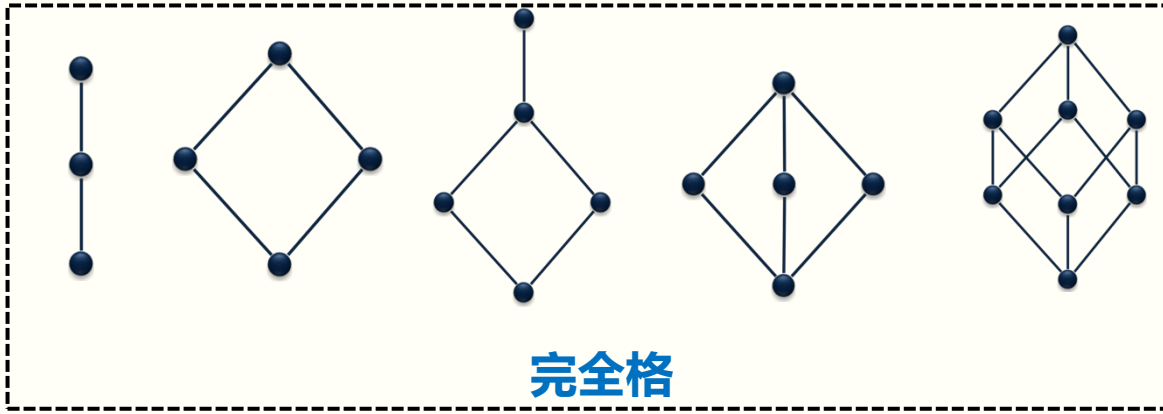
非格



半格



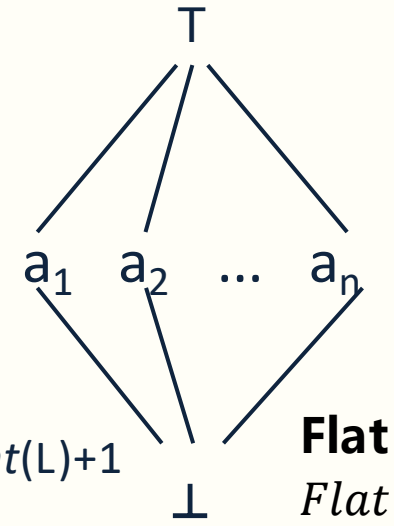
格



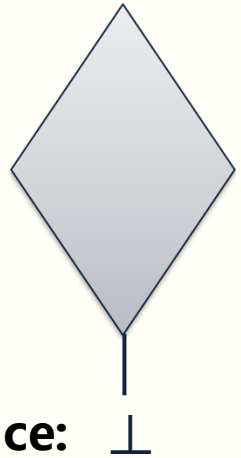
完全格

格（Lattices）

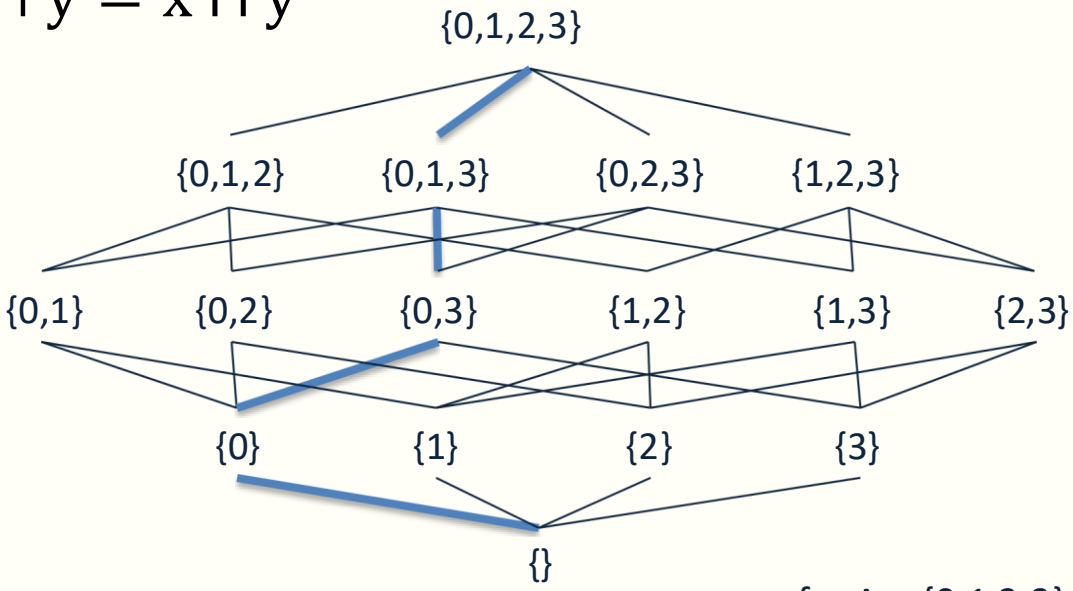
- **height：the length of the longest path in the lattice**
  - For complete lattice: the length from ⊥ to ⊤
- **Powerset lattice:**
  - Powerset of every finite set A defines a complete lattice
  - $(\mathcal{P}(\mathrm{A}), \subseteq), \bot = \emptyset, \top = \mathrm{A}, \mathrm{x} \sqcup \mathrm{y} = \mathrm{x} \cup \mathrm{y}, \mathrm{x} \sqcap \mathrm{y} = \mathrm{x} \cap \mathrm{y}$
- **Flat lattice and Lift lattice**



Lift lattice:
height(*lift*(L)) = *height*(L)+1



Flat lattice:
$Flat(A)$, where A is a set, $height\big(flat(A)\big) = 2$



for A = {0,1,2,3}

格（Lattices）

- **Product lattice:**
  - $L_1 \times L_2 \times \cdots \times L_n = \{(x_1, x_2, \cdots, x_n) \mid x_i \in L_i\}$
  - $L_i (1 \leq \mathrm{i} \leq \mathrm{n})$ is a complete lattice
  - $\sqsubseteq, \sqcup, and \ \sqcap \ are\ computed\ pointwise$
  - $\mathrm{Height}(L_1 \times L_2 \times \cdots \times L_n) = \sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{Height}(L_i)$
- **Map Lattice:**
  - $\boldsymbol{A \rightarrow L = \{[a_1 \mapsto x_1, a_2 \mapsto x_2, \cdots] \mid a_i \in A \wedge x_i \in L\}}$
  - $\sqsubseteq, \sqcup, and \ \sqcap \ are\ computed\ pointwise$
  - $\mathrm{Height}(\mathrm{A} \rightarrow \mathrm{L}) = |\mathrm{A}| \cdot \mathrm{Height}(\mathrm{L})$
- **例子：指针分析**
  - $\boldsymbol{A}$ is the set of program variables
  - $\boldsymbol{L}$ is the powerset of allocation site abstractions (objects)

Andersen's Analysis

- **Key idea: cast as a constraint-solving problem**
  - One subset constraint per instruction
  - Domain abstraction and function abstraction

$$\frac{}{[\![n\colon p := malloc()]\!] \hookrightarrow l_n \in p}\ malloc$$

$$\frac{}{[\![p := \&x]\!] \hookrightarrow l_x \in p}\ address\text{-}of$$

$$\frac{}{[\![p := q]\!] \hookrightarrow p \supseteq q}\ copy$$

$$\frac{}{[\![*p := q]\!] \hookrightarrow *p \supseteq q}\ assign$$

$$\frac{}{[\![p := *q]\!] \hookrightarrow p \supseteq *q}\ dereference$$

- The constraint solver can give the most precise solution.
  - Why? (optional)
  - Constraints are equivalent to abstract functions

$$e \in v \equiv Pts(v) = Pts(v) \cup \{e\}$$
$$\mathrm{x} \supseteq \mathrm{y} \equiv \mathrm{Pts(x)} = \mathrm{Pts(x)} \cup \mathrm{Pts(y)}$$

  - Abstract function is monotone
  - Abstract domain is finite
    - Objects are finite
  - $(\mathrm{Var} \rightarrow \mathcal{P}(\mathrm{Objects}))$ forms a map lattice
  - Abstract function work on this lattice from $\perp$ will reach the least fixed point (e.g., the most precise solution)

Kleene's fixed-point theorem

❑Monotonicity

A function f: L → L (L is a lattice) is monotonic if ∀x, y ∈ L,
x ⊑ y → f(x) ⊑ f(y)

❑Fixed point

$x \in L$ is a fixed point of function $f: L \to L$ iff $f(x) = x$

❑Kleene's fixed-point theorem

In a complete lattice with finite height, every monotone function f has a *unique least fixed-point*:

$$lfp(f) = \bigsqcup_{i \geq 0} f^i(\bot)$$

Proof of existence and uniqueness

❑Existence

- Clearly, $\perp \sqsubseteq f(\perp)$
- Since f is monotone, we also have $f(\perp) \sqsubseteq f^2(\perp)$; By induction, $f^i(\perp) \sqsubseteq f^{i+1}(\perp)$
- This means that $\perp \sqsubseteq f(\perp) \sqsubseteq f^2(\perp) \sqsubseteq \ldots f^i(\perp) \ldots$ is an increasing chain
- L has finite height, so for some k: $f^k(\perp) = f^{k+1}(\perp)$
- If $x \sqsubseteq y$ then $x \cup y = y$, So $lfp(f) = f^k(\perp)$

❑ Uniqueness of least

- Assume that x is another fixed-point: $x = f(x)$
- Clearly, $\perp \sqsubseteq x$
- By induction and monotonicity, $f^i(\perp) \sqsubseteq f^i(x) = x$
- In particular, $lfp(f) = f^k(\perp) \sqsubseteq x$, i.e. $lfp(f)$ is least
- Uniqueness then follows from anti-symmetry
  - Suppose x is another least fixed-point, we have $x \sqsubseteq \mathrm{lfp(f)}$

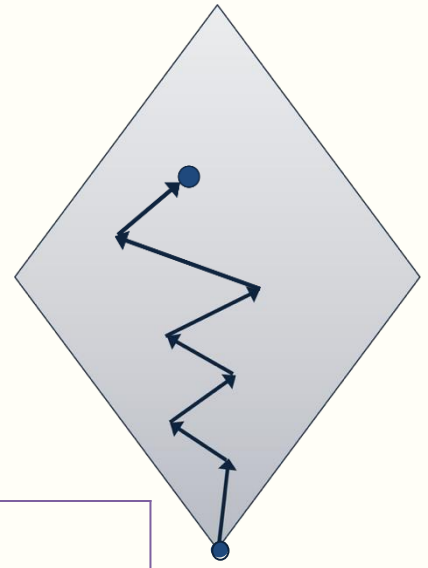
Andersen's Analysis

Let $f_1, f_2 : L \to L$ (L is a lattice) be two monotonic functions, then their **composition** $f_1 \circ f_2$ is also **monotonic**.

Let $f : L \to L$ ($L = (Var \to \mathcal{P}(OBJs))$ is a map lattice) be the constraint solver of Andersen-style pointer analysis, obviously, f is the composition of many constraint-equivalent abstract functions, by Kleene's fixed-point theorem, the solution of Ansersen's analysis is $lfp(f)$.

- **The time complexity of *lfp*(f) depends on:**
  - the height of the lattice
  - the cost of computing f
  - the cost of testing equality

```
x = ⊥ ;
do  {
    t = x;
    x = f(x);
} while  (x≠t);
```



Implementation: TIP/src/tip/solvers/FixpointSolvers.scala

A more efficient implementation

❑The Cubic Framework ($O(n^3)$)

- ➢A set of tokens $\mathrm{T} = \{\mathrm{t}_1, \mathrm{t}_2, \cdots, \mathrm{t}_\mathrm{k}\}$,
- ➢A collection of constraint variables $\mathrm{V} = \{\mathrm{x}_1, \cdots, \mathrm{x}_\mathrm{n}\}$
- ➢A collection of constraints of these forms:

• $t \in x$	;
• $x \subseteq y$	; inclusion constraints
• $t \in x \Longrightarrow \mathrm{y} \subseteq \mathrm{z}$	; conditional constraints

❑Solution: reachability on a constraint graph

- ➢Each variable is mapped to a node
- ➢Each node has a bitvector in $\{0,1\}^k$, initially set to all 0's
- ➢Each bit has a list of pairs of variables, modeling conditional constraints
- ➢The edges model inclusion constraints

求解过程：传播+约束维护

- $x.sol \subseteq T$:
  - the set of tokens for x (the 1's in its bitvectors)
- $x.succ \subseteq V$:
  - the successors of x (the edges)
- $x.cond(t) \subseteq V \times V$:
  - the conditional constraints for x and t
- $W \subseteq T \times V$:
  - a worklist (initially empty)

- $t \in x$
  addToken(t, x)
  propagate()
- $x \subseteq y$
  addEdge(x, y)
  propagate()
- $t \in x \rightarrow y \subseteq z$
  if t ∈ x.sol
    addEdge(y, z)
    propagate()
  else
    add (y, z) to x.cond(t)

```
addToken(t, x):
  if t ∉ x.sol
    add t to x.sol
    add (t, x) to W

addEdge(x, y):
  if x ≠ y ∧ y ∉ x.succ
    add y to x.succ
    for each t in x.sol
      addToken(t, y)

propagate():
  while W ≠ ∅
    pick and remove (t, x) from W
    for each (y, z) in x.cond(t)
      addEdge(y, z)
    for each y in x.succ
      addToken(t, y)
```

$O(n^3)$的时间复杂度，n是变量数量

Andersen's Analysis

❑使用Cubic框架求解

$$\frac{}{[\![n\colon p := malloc()]\!] \hookrightarrow l_n \in p}\ malloc$$

$$\frac{}{[\![p := \&x]\!] \hookrightarrow l_x \in p}\ aaaress\text{-}of$$

$$\frac{}{[\![p := q]\!] \hookrightarrow p \supseteq q}\ copy$$

$$\frac{}{[\![*p := q]\!] \hookrightarrow *p \supseteq q}\ assign$$

$$\frac{}{[\![p := *q]\!] \hookrightarrow p \supseteq *q}\ dereference$$

第一种形式：$t \in x$

第二种形式：$x \subseteq y$

第三种形式：

$\&x \in p \Rightarrow q \subseteq x$

$\&y \in q \Rightarrow y \subseteq p$

➢改进：SCC缩点、类型过滤（eliminate type-incompatible Objects）

❑理论复杂度$O(n^3)$, 实际复杂度$O(n^2)$（Sridharan et al. [SAS,09]）

Andersen's Analysis

❑一些扩展

- ➢域敏感（field sensitivity）
  - ○ 域不敏感： treats fields `.f` as dereferences `*`
  - ○ 区分不同的域（field），在Java指针分析中很重要

$$\frac{}{[\![p := q.f]\!] \hookrightarrow p \supseteq q.f}\ \textit{field-read}$$

$$\frac{}{[\![p.f := q]\!] \hookrightarrow p.f \supseteq q}\ \textit{field-assign}$$

c/c++转化成第二种形式：

$$x \subseteq y$$

Java转化成第三种形式(p.f相当于c/c++中的(*p).f)：

$$0 \in q \Rightarrow 0.\mathrm{f} \subseteq p$$

$$0 \in p \Rightarrow q \subseteq 0.\mathrm{f}$$

- ➢支持函数调用
  - ○ 参数和返回值的传递建模成copy语句

```
r = m(a1, ..., an)
```

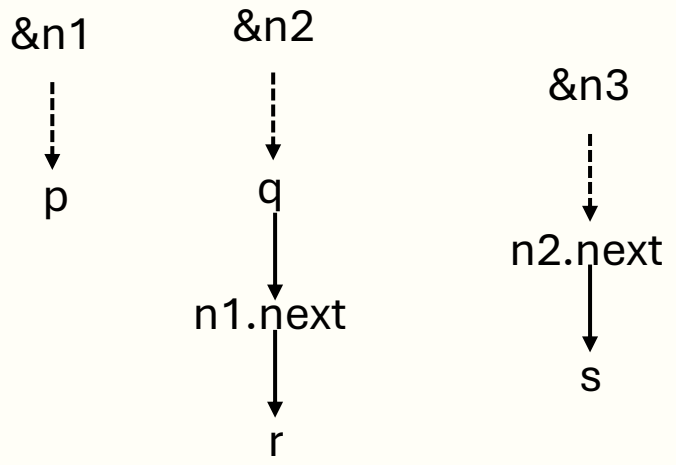
```
T m(T p1, ... T pn) {
  ...
  return ret;
}
```

$$\frac{}{[\![\mathrm{r} = \mathrm{m}(\mathrm{a1}, \cdots, \mathrm{an}]\!] \hookrightarrow \mathrm{p1} \supseteq \mathrm{a1}, \ldots, \mathrm{pn} \supseteq \mathrm{an}, \mathrm{r} \supseteq \mathrm{ret}}\ \textit{call}$$

Example Program

```c
#include <stdio.h>
struct Node { struct Node *next; };
int main() {
    struct Node n1, n2, n3;
    struct Node *p = &n1;
    struct Node *q = &n2;
    n1.next = q;
    struct Node *r = p->next;
    n2.next = &n3;
    struct Node *s = r->next;
    return 0;
}
```

Constraint Graph (so-called Pointer Assignment Graph)



Vars	PTS	Vars	PTS
p	{&n1}	n2.next	{&n3}
q	{&n2}	s	{&n3}
n1.next	{&n2}	r	{&n2}

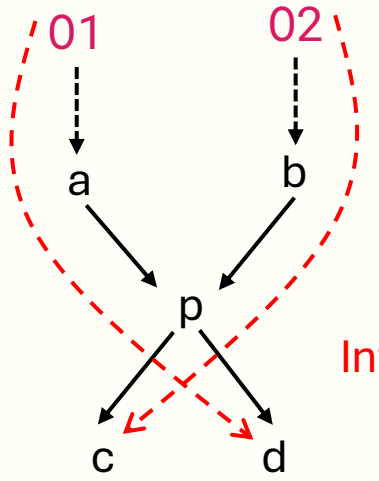
留了个更复杂的例子作为可选课后作业

Context Sensitivity

```
1. #include <stdlib.h>
2. int *id(int *p) {
3.     return p;
4. }
5. int main() {
6.    int *a, *b, *c, *d;
7.    a = (int *)malloc(sizeof(int)); // O1
8.    c = id(a);
9.    b = (int *)malloc(sizeof(int)); // O2
10.   d = id(b);
11.   return 0;
12. }
```

- Andersen's analysis is context-insensitive
  - Joins information across callsits to same function
  - Loses precision due to modeling infeasible paths
  - Can we "remember" where to return?

Constraint Graph (so-called PAG)



Infeasible value flow paths

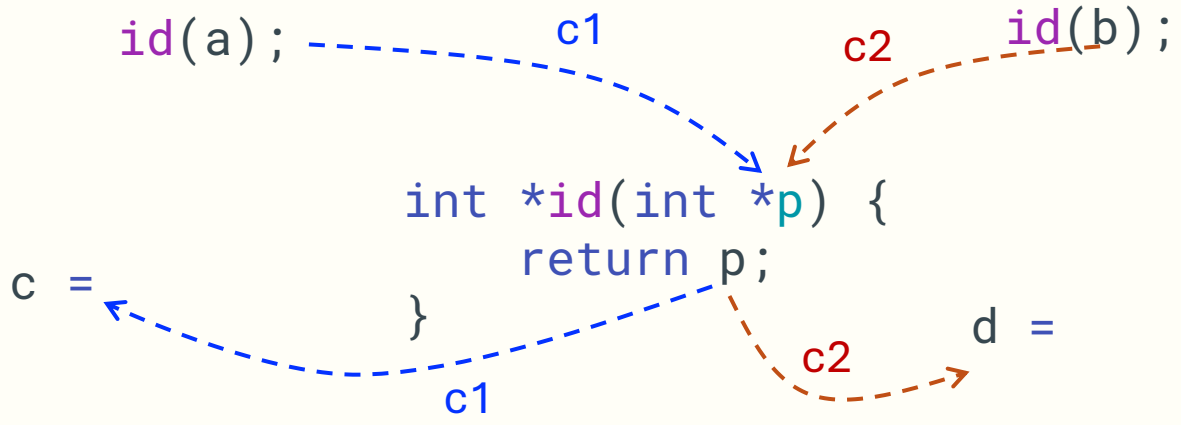
Vars	PTS	Vars	PTS
a	{O1}	b	{O2}
p	{O1, O2}		
c	{O1, O2}	d	{O1, O2}

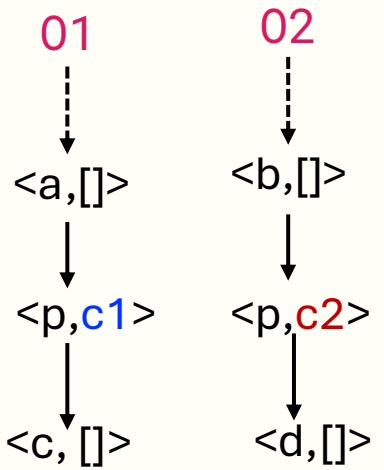
Spurious Points-to Objects

Context Sensitivity

❑Key idea:

- ➢Separate analyses for functions called in different "contexts"
- ➢"context": an abstract concept that is statically definable; used to differentiate different calls to a function
- ➢Contexts are used to decorate graph nodes





Precise!

Vars	PTS	Vars	PTS
<a, []>	{O1}	<b, []>	{O2}
<p, c1>	{O1}	<p, c2>	{O2}
<c, []>	{O1}	<d, []>	{O2}

Types of Context Sensitivity

- No context sensitivity, []
- Callsite sensitivity
  - Call strings, $[l_1, \cdots, l_n]$
  - l_i: the line label of the callsite
- Object sensitivity, $[o_1, \cdots, o_n]$
  - o_i is an allocation site
- Type sensitivity, $[t_1, \cdots, t_n]$
  - t_i is the type of an allocation object
- Value contexts
  - States (environment) at the given callsite
- $k-$ limiting technique
- Unscalable when $k \geq 3$ in practice

```
1. int *id(int *p) {
2.   return p;
3. }
4. int *wid_1(int *p_1) {
5.   return id(p_1);
6. }
7. ...
8. int *wid_n(int *p_n) {
9.   return wid_(n-1)(p_n);
10. }
11. c = wid_n(a);
12. d = wid_n(b);
```

Two contexts for differentiate id():

[l5, …, l9, l11]
[l5, …, l9, l12]

Context length: n+1

```
int factorial(int* n) {
  if (*n == 0) {
    return 1;
  } else {
    int t = *n;
    *n = t - 1;
    return t * factorial(n)
  }
}
int x = 100;
int y = factorial(&x);
```

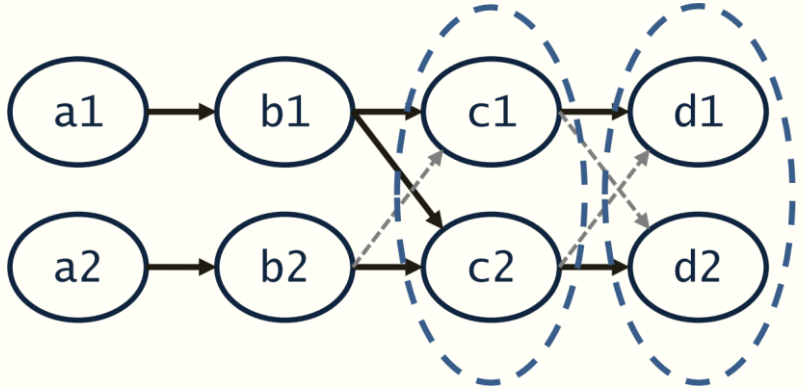
Context length could be infinite.

Steensgaard's Analysis

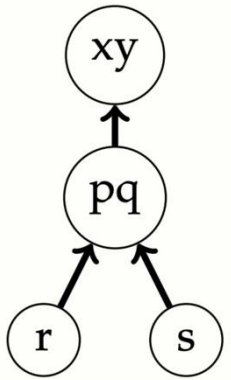
- **Problem:** Quadratic-in-practice is still not ultra-scalable
  - **Want a faster algorithm! Need ~LINEAR. How?**
- **Challenge:**
  - Solution space of pointer analysis (e.g. points-to sets) is $O(n^2)$
- **Key idea:**
  - Use constant-space per pointer.
  - Merge aliases and alternates into the same equivalence class.
  - p can point to q or r? Let's treat q and r as the same pseudo-var and merge everything we know about q and r.
  - Points-to "sets" are basically singletons
- **Algorithm runs in $O(n * \alpha(n))$ using union-find structure**
  - Almost linear, very scalable in practice (Millions of LoC)
  - Yes, very imprecise!

Steensgaard's Analysis-Examples

```
a1 = &b1;
b1 = &c1;
c1 = &d1;
a2 = &b2;
b2 = &c2;
c2 = &d2;
b1 = &c2;
```



$$
\begin{aligned}
1: &\quad p := \&x \\
2: &\quad r := \&p \\
3: &\quad q := \&y \\
4: &\quad s := \&q \\
5: &\quad r := s
\end{aligned}
$$



Steensgaard's Analysis

$$\frac{}{[\![p := q]\!] \hookrightarrow join(*p, *q)}\ copy$$

$$\frac{}{[\![p := \&x]\!] \hookrightarrow join(*p, x)}\ address\text{-}of$$

$$\frac{}{[\![p := *q]\!] \hookrightarrow join(*p, **q)}\ dereference$$

$$\frac{}{[\![*p := q]\!] \hookrightarrow join(**p, *q)}\ assign$$

```
join(ℓ1, ℓ2)
    if (find(ℓ1) == find(ℓ2))
        return
    n1 ← *ℓ1
    n2 ← *ℓ2
    union(ℓ1, ℓ2)
    join(n1, n2)
```

- **Points-to "sets" are basically singletons**

时间复杂度$O(n * \alpha(n))$ (almost linear)，空间复杂度$O(n)$.

（Optional）作业：指针分析应用

❑用Andersen算法分析如下代码

```
#include <stdio.h>
struct Node { struct Node *next; int *data; };
int main() {
  int a, b, c;  struct Node n1, n2, n3;
  int *pa = &a; int *pb = &b; int *pc = &c;
  struct Node *p = &n1;
  struct Node *q = &n2;
  struct Node *r = &n3;
  n1.next = &n2; q->next = r;
  p->data = pa; n2.data = &b
  struct Node *x = p->next;
  struct Node *y = x->next;
  int *d1 = p->data; int *d2 = q->data;
  r->data = pc;
  y = p->next;
  return 0;
}
```

❑用Steensgaard算法分析

```
#include <cstdlib>
struct Node { struct Node *next; };
int main() {
  struct Node *a, *b, *c, *tmp;
  a = (struct Node *)malloc(sizeof(struct
Node));
  b = (struct Node *)malloc(sizeof(struct
Node));
  c = (struct Node *)malloc(sizeof(struct
Node));
  tmp = a;
  tmp = b;
  a = c;
  a->next = b;
  b->next = c;
  return 0;
}
```

（Optional）项目：熟悉指针分析实现

- **Qilin指针分析框架**
  - https://github.com/QilinPTA/Qilin
  - 支持多种指针分析优化技术
- **任务：**
  - 选一组Java程序（3-4个即可），尝试用Qilin框架的技术进行指针分析
  - 阅读源码，探究Qilin是怎么支持多种上下文敏感性的？
  - 指出Qilin框架设计和实现上可以改进和优化的地方？
  - 写一个1000字左右的阅读报告
  - 语言要求言简意赅，不无病呻吟
  - 可以使用示意图等辅助说明